

## Retarded $N$ -Body Dispersion Potential: Molecular QED Formulation

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A well-known success of the theory of non-relativistic quantum electrodynamics (QED) [1,2] is the calculation of the dispersion potential between two particles separated by a distance  $R$ , by Casimir and Polder [3]. They showed that accounting for the finite speed of propagation of electromagnetic signals weakened the energy shift by a factor of  $R^{-1}$  relative to the  $R^{-6}$  London dispersion formula. Aub and Zienau [4] later evaluated the leading non-additive term arising from three-bodies, extending the familiar Axilrod-Teller-Muto triple dipole dispersion potential [5,6], which was found to be the near-zone limit of the result valid for all separations.

We present an expression, obtained using molecular QED, for the retarded  $N$ -body dispersion energy shift for atoms or molecules with arbitrary electric multipole polarizability, generalizing a formula that was limited to the electric dipole approximation [7]. Higher-order many-body contributions are now featuring in the interactions between rare gases, alkali and alkaline Earth metal atoms, and combinations thereof [8]. A response theory formulation is adopted in which each species in turn responds through its generalized electric multipole polarizability to the electric displacement field of the remaining  $N-1$  bodies. This approach first entails the solution of the source dependent wave equation for the displacement field [9]. The formula derived is shown to agree with previously obtained generalized expressions for two-[10] and three-[11] body dispersion potentials between species with arbitrary electric multipole polarizable characteristics.

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